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Opinion formation in time-varying social networks: The case of Naming Game

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We study the dynamics of the Naming Game as an opinion formation model on time-varying social networks. This agent-based model captures the essential features of the agreement dynamics by means of a memory-based negotiation process. Our study focuses on the impact of time-varying properties of the social network of the agents on the Naming Game dynamics. We investigate the outcomes of the dynamics on two different types of time-varying data - (i) the networks vary across days and (ii) the networks vary within very short intervals of time (20 seconds). In the first case, we find that networks with strong community structure hinder the system from reaching global agreement; the evolution of the Naming Game in these networks maintains clusters of coexisting opinions indefinitely leading to metastability. In the second case, we investigate the evolution of the Naming Game in perfect synchronization with the time evolution of the underlying social network shedding new light on the traditional emergent properties of the game that differ largely from what has been reported in the existing literature.

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I. INTRODUCTION

Social networks are inherently dynamic. Social interactions and human activities are intermittent, the neighborhood of individuals moving over a geographic space evolves over time, links appear and disappear in the World-Wide-Web. The essence of social network lies in its time-varying nature. Links may exist for a certain time period and may be recurrent. In summary, as time progresses, the societal structure keeps changing. Similarly, with the evolution of time, social conventions, shared cultural and linguistic patterns reshape themselves. Opinions spread, some gets trapped into communities, some crosses the barrier of local groups/communities and become accepted globally among different communities and some die competing with others. Most of these social phenomena can be modeled and analyzed in a time-varying framework. Almost all previous work is limited to the analysis of the Naming Game dynamics on static networks [3, 5, 7, 9–11, 16, 18, 27]. Therefore, in this paper, we focus on the competing opinion formation over time-varying real-world social networks. One way of viewing at time-varying networks is as a series of static graphs accumulated over a fixed time interval; however this kind of networks do not always perfectly capture the temporal ordering of the links appearing in the system which may sometimes lead to over/under-estimation of network topologies. Thus, we plan to investigate the opinion formation process on both the accumulated static graphs as well as on its detailed time-resolved counterpart.

In this paper, we focus on the basic Naming Game model (NG) [5] to study how opinions spread with time and how societies move towards consensus in the adoption of a single opinion through negotiation or agree upon multiple opinions due to non-uniform interaction pattern among different communities. The evolution of the system in this model takes place through the usual local pairwise interactions among artificial agents that necessarily captures the generic and essential features of an agreement process. This model was expressly conceived to explore the role of self-organization in the evolution of languages [24, 25] and has acquired a paradigmatic role in semiotic dynamics that studies evolution of languages through invention of new words, grammatical constructions and more specifically, through adoption of new meaning for different words. NG finds wide applications in various fields ranging from artificial sensor network as a leader election model [2] to the social media as an opinion formation model.

The minimal Naming Game (NG) consists of a population of N agents observing a single object in the environment (may be a discussion on a particular topic) and opining for that by means of communication to one another through pairwise interactions, in order to reach a global agreement. The agents have at their disposal an internal inventory, in which they can store an unlimited number of different words or opinions. At the beginning, all the individuals have empty inventories. At each time step, the dynamics consists of a pairwise interaction between randomly chosen individuals. The chosen individuals can take part in the interaction as a "speaker" or as a "hearer." The speaker voices to the hearer a possible opinion for the object under consideration; if the speaker does not have one, i.e., his inventory is empty, he invents an opinion. In case where he already has many opinions

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stored in his inventory, he chooses one of them randomly. The hearer's move is deterministic: if she possesses the opinion pronounced by the speaker, the interaction is a "success", and in this case both speaker and hearer retain that opinion as the right one, removing all other competing opinions/words in their inventories; otherwise, the new opinion is included in the inventory of the hearer, without any cancellation of opinions in which case the interaction is termed as a "failure" (see fig 1). The game is played on a fully connected network, i.e., each agent can, in principle, communicate with all the other agents, and makes two basic assumptions. One assumes that the number of possible opinions is so huge that the probability of a opinion being reinvented is practically negligible (this means that similar opinions is not taken into account here, although the extension is trivially possible). As a consequence, one can reduce, without loss of generality, the environment to be consisting of only one single object/topic of discussion.

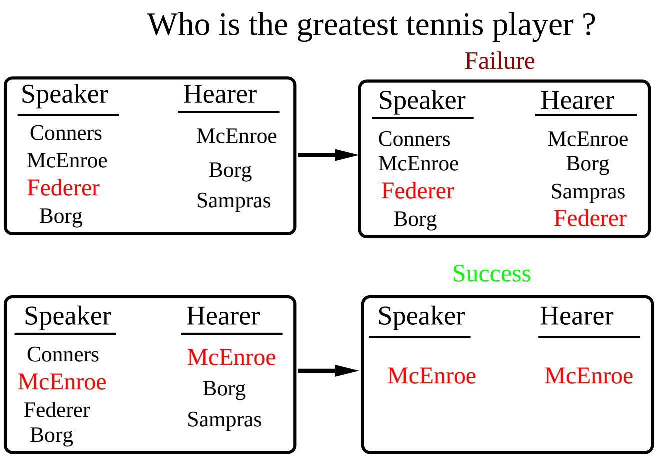

FIG. 1: (Color online) Agent's interaction rules in basic NG. Suppose there is a topic on which a discussion is going on, say "Who is the greatest tennis player?". (Top) The speaker chosen at random, opines for "Federer" (also chosen randomly from his inventory of opinions). Now, the hearer (again chosen at random) does not have this opinion in her inventory, and therefore she adds the opinion "Federer" in her inventory and the interaction is a failure . (Bottom) The speaker opines for "McEnroe" and in this case the opinion is present in the hearer's inventory. So, they delete all other opinions except "McEnroe". The interaction this time is "success".

Although the system reaches a global consensus through the invention and decay of opinions, it is interesting to note the important differences from other opinion formation models. In Axelrods model [1], each agent is endowed with a vector of opinions, and can interact with other agents only if their opinions are already close enough; in Sznajds model [26] and in the Voter model [15], the opinion can take only two discrete values, and an agent takes deterministically the opinion of one of its neighbors. Further in [12], the opinion is modeled as a unique variable and the evolution of two interacting agents is deterministic. In the Naming Game model on the other hand, each agent can potentially have an unlimited number of possible discrete states (or opinions) at the same time, accumulating in its memory different possible opinions; the agents are able to "wait" before reaching a decision. Moreover, each dynamical step can be seen as a negotiation between a speaker and a hearer, with a certain degree of stochasticity.

In this paper, we consider the NG dynamics on two different types of time-varying data; one varying across days while another varying over very short intervals of time (20 seconds). In the first case, we observe that networks with strong community structures delay the convergence due to co-existence of competing and long-lasting clusters of opinions. In the second case, the games are played in perfect synchronization with the time-evolution of the network. In this case, we observe that the global observables are markedly different from the case where the games are played on the static (and composite) version of the same network as well as from the traditional results reported in the literature.

The rest of the paper is organized as follows. Section 2 is devoted for the discussion of the state of the art. In Section 3, we describe the datasets on which we investigate the Naming Game dynamics in a time-varying social scenario. Section 4 provides the elaborate model description. In Section 5, we present the results and provide explanations for our findings. Finally, conclusions are drawn in section 6.

II. RELATED WORK

Most previous studies of the NG model in semiotic/opinion dynamics have focused on populations of agents in which all pairwise interactions are allowed, i.e., the agents are placed on the vertices of a fully connected graph. In statistical mechanics, this topological structure is commonly referred to as mean-field topology [5, 6]. Apart from mean-field case, the model has also been studied on regular lattices [3, 18]; small world networks [7, 10, 16, 18]; random geometric graphs [14, 17, 18]; and static [9, 11, 27], dynamic [21], and empirical [19] complex networks.

Lu et. al. [19] have studied the Naming Game dynamics on a high-school friendship network and have shown that the presence of community structures affect the behavior of the dynamics through the formation of long-living late-stage meta-stable clusters of opinions. Therefore, they propose injection of committed agents (agents who never change their opinion) into the population for fast agreement of the dynamics. Nardini et. al. [21] studied the dynamics of NG on adaptive networks where the connections can be rewired with the evolution of the game.

All these prior works have studied the NG dynamics on essentially static networks. Therefore, our study reported here is unique and different from the literature since we

consider the evolution of the NG dynamics over time-varying social structure.

III. DATASETS

For the purpose of the investigation of the NG dynamics on time-varying networks, we consider two specific real-world face-to-face contact datasets and present our results on each of them. Both the datasets are obtained from http://www.sociopatterns.org/datasets/. The first dataset we consider is the face-to-face interaction data of visitors of the Science Gallery in Dublin, Ireland during the spring of 2009 at the event of art-science exhibition “INFECTIOUS: STAY AWAY”. The dataset contains the cumulative daily networks of the visitors for sixty-nine days [13]. The nodes represent visitors of the Science Gallery while the edges represent close-range face-to-face proximity (measured using RFID devices carried by each visitor) between the concerned persons. The weights associated with the edges are the number of 20 seconds intervals during which close-range face-to-face proximity could be detected. Thus, these daily networks can be thought of as sixty-nine snapshots of a time-varying societal structure with a periodicity of 24 hours. We will refer to this as the SG dataset.

We also consider the time-resolved datasets which are the dynamic counterparts of the daily cumulated contact networks of the SG dataset. From the 69 daily instances, we consider time-resolved contact pattern of four instances day 9, 20, 22 and 26, which can be considered as the representatives of all the instances. These time-resolved data are refered to as SGD dataset.

The last dataset we consider is the face-to-face interaction data of the conference attendees of the ACM Hypertext 2009 conference held in Institute for Scientific Interchange Foundation in Turin, Italy, from June 29th to July 1st, 2009, where the SocioPatterns project deployed the Live Social Semantics application. The dataset contains the dynamical network of face-to-face proximity of 115 conference attendees over about 2.5 days. In future reference, we will refer to this as the HT dataset.

IV. THE MODEL DESCRIPTION

The basic NG Model can be summarized as follows. At each time step ($t = 1, 2, ..$) two agents are randomly selected to interact: one of them plays the role of speaker, the other one that of hearer. The interactions obey the following rules

- The speaker voices an opinion from its list of opinions to the hearer. (If the speaker has more than one opinion on his list, he randomly chooses one; if he has none, he invents one randomly.)
- If the hearer has this opinion, the communication is termed “successful”, and both players delete all other opinions, i.e., collapse their list of opinions to this one opinion. Therefore, they meet a local agreement.
- If the hearer does not have the opinion transmitted by the speaker (termed “unsuccessful” communication), she adds the opinion to her list of opinions without any deletion.

Note that in this model any agent is free to interact with any other agent, i.e., the underlying social structure is assumed to be fully connected. For the purpose of our analysis however, we assume that the agents are embedded on realistic social networks (i.e., SG and HT) that are continuously varying over time. In this case, although the basic rules of the game remain exactly same, the only issue is to devise a strategy for the speaker-hearer selection. We consider two variants of this selection, the first one being suitable for the SG dataset and the second one for the SGD and HT dataset.

Strategy I: Here we randomly select a speaker and preferentially choose a hearer among its neighbors. Our intention here is to simulate an important criterion: we talk most preferably to those with whom we had already met before. This is implemented as follows:

- The speaker i is selected randomly.
- The hearer j is selected using the preferential rule, with the probability

  $$p_{ij} = \frac{w_{ij}}{\sum_{j=1}^{k} w_{ij}}$$

  where w_{ij} can be thought of as the total number of contact events between the pair i and j while k is the degree of agent i (i.e, the number of other agents that i is connected to at a particular instant of time).

Strategy II: This variant is quite straight-forward. We choose a random speaker and a random hearer among its neighbors to impart the equal importance of each pair of connections.

The main quantities of interest which describe the emergent properties of the system are

- the total number $N_w(t)$ of words/opinions in the system at the time t (i.e., the total size of the memory);
- the number of different words/opinions $N_d(t)$ in the system at the time t;
- the average success rate $S(t)$, i.e., the probability, computed averaging over many simulation runs, that the chosen agent gets involved in a successful interaction at a given time t.

From a global perspective, the quantities which are of interest are the time to reach the global consensus (t_{conv}), the maximum memory required by the agents during the process (N_w^{max}) and the time required to reach the memory peak (t_{max}).

V. RESULTS AND DISCUSSIONS

In this section, we present the results of the analysis of the NG dynamics on the SG, SGD and the HT datasets.

A. Analysis on day-wise SG dataset

We have studied the opinion formation process on the sixty-nine days of close interactions among the visitors for the SG dataset. The primary focus of this study is to find the behavior of the global quantities of the NG dynamics with the evolution of the topology over time. We play the NG on each of these daily networks of sixty-nine days following Strategy I. The networks are not always fully connected. In case of disconnected components, N_d should never converge to 1 and consequently, the emergence of multi-opinion state is observed. Therefore, in this case we redefine t_{conv} as the time to reach the following state: $N_w = N$ and $N_d = c$ where c is the number of disconnected components. The natural question that arises is how the opinion dynamics gets affected as the underlying network structure varies over the days. It is interesting to note that the memory peak N_w^{max} and the time to reach the memory peak t_{max} have a strong correlation with the system size N. In fact, they have a linear scaling with the population size N (See fig 2). This relationship is in agreement with what has been observed in the literature for small-world, scale-free and random networks [7, 11] where both N_w^{max} and t_{max} scales as $O(N)$. Nevertheless, these cumulative daily networks do not resemble any of the well-known topologies which will be clear when we dig into the results of t_{conv}. We observe that the behavior of t_{conv} (see fig 3(a), (b)) is not in lines of the existing literature where it is usually noted that $t_{conv} \sim N^{1.4}$. Therefore the natural question that needs to be addressed is that what is (are) the property (s) of the underlying network that leads to such a nonconforming behavior of t_{conv}.

In fact, the answer to this question lies in the common behavior of the real-world social networks. These networks typically consist of a number of communities; nodes within communities are more densely connected, while links bridging communities are sparse. The effect of the community structure plays a dominant role with the emergence of long-lasting multi-opinion states at the late stage of the dynamics which has also been observed in [11] and [19]. In fact, each community reaches internal consensus fast but the weak connections between communities are not sufficient for opinions to propagate from one community to the other leading to long multi-opinion states which are also known as "metastable states" in the domain of statistical physics. Formally, a metastable state is a state of the dynamics where global shifts are always possible but progressively more unlikely and the response properties depend on the age of the system [20]. Community structures are essentially authentic signatures of metastability which inhibits the dynamics leading to very slow convergence.

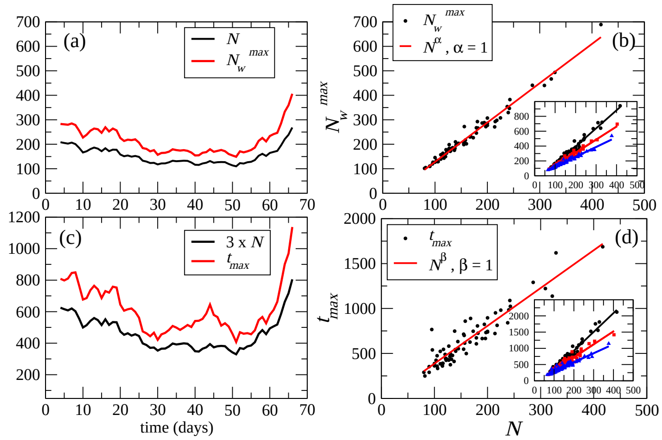

FIG. 2: (Color online) Scaling relation of N_w^{max} and t_{max} with population size N. (a) Temporal behavior of N_w^{max} and the population size N for each of the 69 instances. Data are smoothed by taking 7 point running average. (b) Scaling of N_w^{max} with N which is linearly fitted. The inset shows the scaling of N_w^{max} in the thresholded networks where the threshold is set to 1, 2 and 5 respectively. All these curves are linearly fitted. (c) Variation of t_{max} and population size N with time. Data are smoothed by taking 7 point running average. (d) Scaling of t_{max} with N which is linearly fitted. The inset shows the scaling of t_{max} in the thresholded networks where the threshold is set to 1, 2 and 5 respectively. All these curves are linearly fitted.

Presence of community structures slows down the dynamics, however, what renders the system even slower is the presence of different-sized communities. The reason for this is quite straight-forward: the agents that are part of a larger size community have a higher probability of being chosen for a game than those belonging to a smaller size community. This is a reminiscent of the fact that the agents are chosen randomly which automatically increases the chances of landing in a larger size community simply because a larger bulk of the population is confined within this community. Therefore, even when consensus is reached very fast in a large community, the system keeps on choosing agents from this community itself mostly resulting in "success with no outcome". Further, since the inter-community links are weak, and agents from smaller communinities are hardly chosen the overall state of the system hardly changes thereby always keeping the agents away from the global consensus. This is reflected through fig 3(c), (d) and (e) where we report the correlation of t_{conv} with the variance of the community sizes. The basic idea is as follows: if a network gets decomposed into m communities each of size $s_1, s_2, ..., s_m$ then we calculate the statistical variance of this size distribution and plot it against t_{conv}. For the purpose of community analysis, we use three standard algorithms - Newman and Girvan (NGR) [23], Newman, Clauset and Moore (NCM) [8] and community detection by eigen vec-

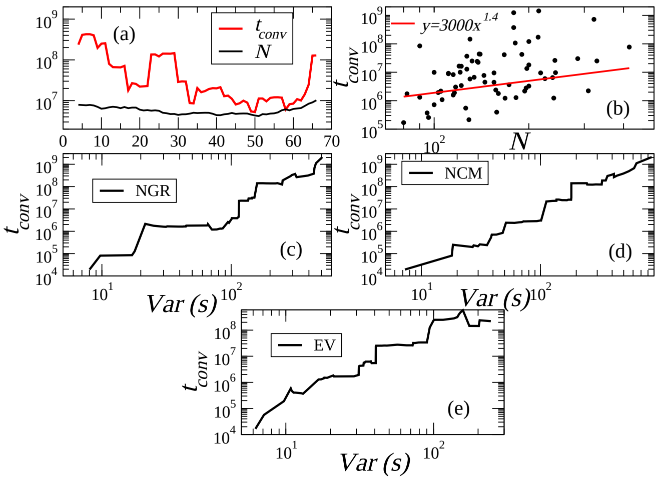

FIG. 3: (Color online) (a)Temporal behavior of t_{conv} and the population size N. Data are smoothed by taking 7 point running average. (b) Scatter plot of t_{conv} and N which could not be fitted with $y = x^{1.4}$ ($R^2 \approx 0.03$). (c), (d) and (e) Correlation of t_{conv} with variance of community sizes $Var(s)$ detected by various community detection algorithms. The curves are smoothed by taking 20 point running averages.

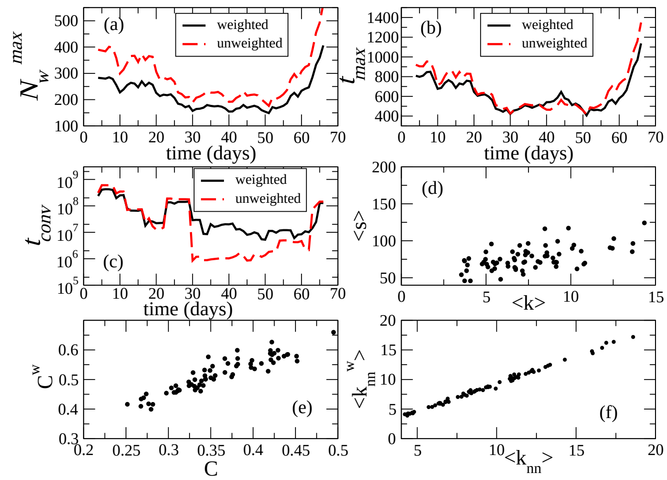

FIG. 4: (Color online) Effect of edge weights on the dynamics. (a), (b) and (c) Temporal evolution of N_w^{max}, t_{max} and t_{conv} for the weighted and unweighted NG respectively smoothed over a time sliding window of size 7 . (d) Variation of $\langle k \rangle$ with $\langle s \rangle$. (e) Variation of C^w with C. (f) Variation of $\langle k_{nn}^w \rangle$ with $\langle k_{nn} \rangle$.

tor (EV) [22] and in each case we observe that t_{conv} has a strong positive correlation with the variance of the community sizes (see fig 3 (c), (d) and (e)).

1. *Effect of edge weights*

As we have suggested earlier, we consider two variants of pair selection, the weighted and the unweighted one. In this subsection, we attempt to study the effect of edge-weights on the dynamics. The edge weights play significant role in pair-selection and so there is a possibility that this affects the dynamics. However, what we observe here is in the contrary. The global quantities of interest in case where all the neighboring agents are given equal preference remain roughly equivalent to the case where the weights are considered (see fig 4(a), (b) and (c)). The reason behind this is the skewed distribution of edge weights.

We find that above 60 % edges on average are low-weight edges which somehow drives the dynamics of the preferential model towards the behavior close to the dynamics of the unweighted NG dynamics (see table I). In addition, we also observe a strong correlation between the average degree $\langle k \rangle$ and the average strength $\langle s \rangle$ (see fig 4(d)). The weighted clustering coefficient C^w is also close to the topological clustering coefficient C (see fig 4(e)). Further, the weighted average nearest neighbor degree $\langle k_{nn}^w \rangle$ and the unweighted average nearest neighbor degree $\langle k_{nn} \rangle$ are perfectly correlated (see fig 4(f)).

TABLE I: Distribution of edge weights averaged over all 69 instances.

Edge Weights	% of edges having that weight
1	0.41
2	0.13
3	0.06
4	0.04
5	0.03
Others	0.33

2. *Examples of individual instances*

In this subsection, we dig deeper into the individual snapshots to have a more clear understanding of the ongoing dynamics. From the sixty-nine instances, we present four representative cases that roughly capture all the different characteristics found across the instances. Two among these, consist of disconnected components while the other two are single connected components. Further, two of them (one connected and the other disconnected) show fast convergence while another two (again one connected and the other disconnected) show slow convergence triggered by the presence of community structures leading to metastability. Here we propose two metrics to capture the two distinct behaviors of the convergence time. The first one is the average unique words per community which is denoted by $U(t)$ and defined as follows:

$$U(t) = \frac{\sum_{i=1}^{C} |A_i|}{C}$$

where C is the number of communities and A_i is the list of unique words in community i.

The second metric we propose is the average overlap of unique words across communities which is denoted by $O_c(t)$ and defined as follows:

$$O_c(t) = \frac{2}{C(C-1)} \sum_{i>j} \frac{2(|A_i \bigcap A_j|)}{\sqrt{2(|A_i|^2 + |A_j|^2)}}$$

We consider the daily networks of 9th, 20th, 22nd and 26th day. The 9th day and 22nd day network structures consist of a single connected component with 200 and 240 nodes respectively while the 20th and 26th daily network consist of multiple disconnected components with 96 and 156 nodes respectively. The evolution of $N_w(t)$ shows a steady growth signifying inventions of new opinions coupled with a series of failure interactions until the maxima is reached (see fig 5(a)). From this point onward, the reorganization phase commences and the players encounter mostly successful interaction resulting in the drop of $N_w(t)$ (fig 5(a)). While for the 20th (disconnected network) and 22nd (connected network) day consensus is reached fast, for the 9th (connected network) and the 26th (disconnected network) day the system gets arrested in a long plateau indicating the presence of metastability and strong community structures. The growth of $N_d(t)$ also signifies similar pattern, steady rise followed by steady fall and a plateau (signifying a strong community structure) in case of the 9th and 26th day (see fig 5(b)). To explore the flat plateau region further we report $U(t)$ and $O_c(t)$ in fig 5(c) and 5(d) respectively. It is interesting to note that both $U(t)$ and $O_c(t)$ show a plateau in case of the 9th and the 26th day which is a signature of the fact that the games played in the plateau region predominantly produces success with no deletion of opinions leading to the emergence of metastability.

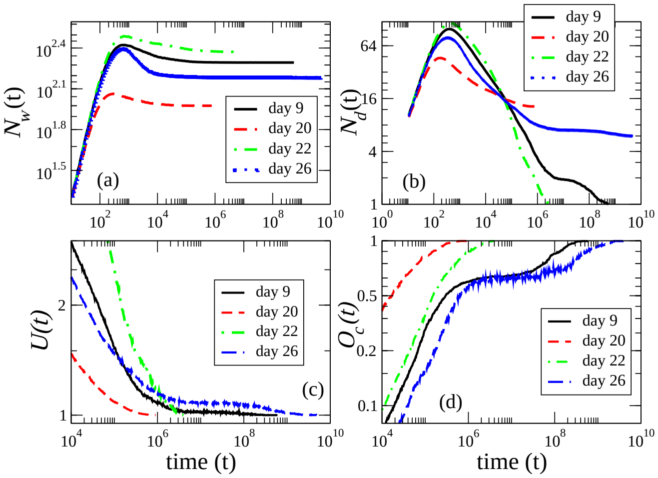

FIG. 5: (Color online) (a) and (b) Comparison of the evolution of the total number of words $N_w(t)$ and number of different words $N_d(t)$ with time on four representative networks. (c) Average number of unique words per community $U(t)$ evolving over time. (d) Temporal evolution of average overlap of words across communities $O_c(t)$. Each point in the above curves represents the average value obtained over 100 simulation runs.

B. Analysis on the time-resolved dataset

In this section, we consider the datasets containing dynamic face-to-face interactions. We play the Naming Game on these time-varying networks in complete synchronization with the real time, i.e., a single game is played on a single time-resolved snapshot of the same network. Thus, at each time step $t = 1, 2 \ldots$, the game is played among those agents that are alive at that particular instant of time in the network. We consider the Strategy II where at each time step, we choose a random speaker and a random hearer among its neighbors.

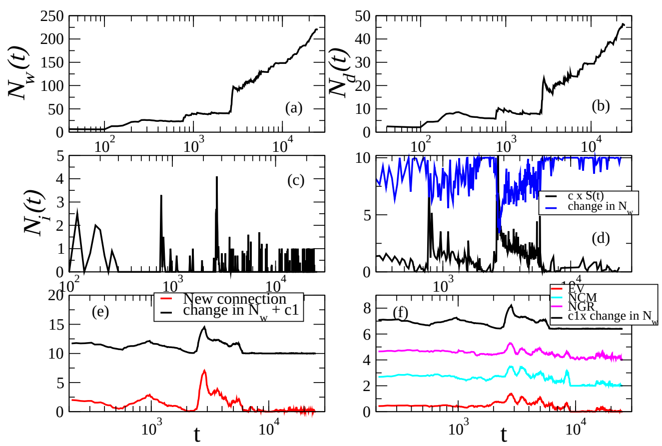

FIG. 6: (Color online) (a) and (b) The temporal evolution of $N_w(t)$ and $N_d(t)$ on time-varying day 9 SGD dataset. The data are averaged over 100 simulation runs. (c) The behavior of the number of inventions of opinions $N_i(t)$ over time. (d) Comparison of $\Delta N_w(t)$ with success rate $S(t)$. (e) Comparison of temporal evolution of $\Delta N_w(t)$ and number of new connections smoothed by taking 20 point running average. (f) Comparison of $\Delta N_w(t)$ with the variance of community sizes (found by NGR, NCM and EV algorithm) evolving over time (the curves are suitably scaled by some constant for the purpose of better visualization). The data are smoothed by taking 20 point running average.

1. *Results from SGD dataset*

In this section, we consider time-resolved dataset of four representatives from the SG dataset. The networks on 9th and 22nd day consist of a single components; however, the 20th and 26th day networks show existence of multiple disconnected components. We analyze each of these time-evolving networks and report the behavior of

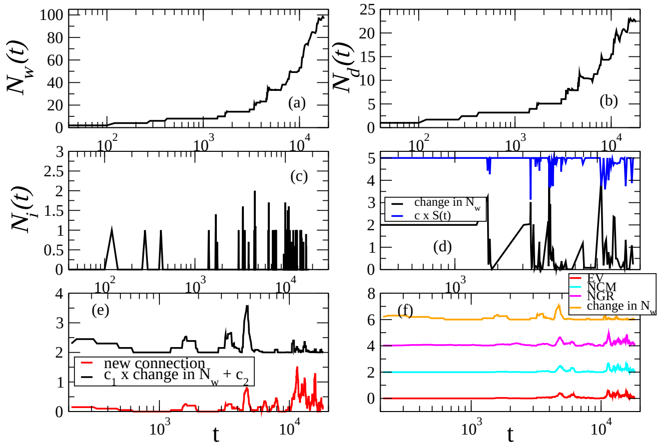

FIG. 7: (Color online) (a) and (b) The temporal evolution of $N_w(t)$ and $N_d(t)$ on time-varying day 20 SGD dataset. The data are averaged over 100 simulation runs. (c) The behavior of the number of inventions of opinions $N_i(t)$ over time. (d) Comparison of $\Delta N_w(t)$ with success rate $S(t)$. (e) Comparison of temporal evolution of $\Delta N_w(t)$ and number of new connections smoothed by taking 20 point running average. (f) Comparison of $\Delta N_w(t)$ with the variance of community sizes (found by NGR, NCM and EV algorithm) evolving over time (the curves are suitably scaled by some constant for the purpose of better visualization). The data are smoothed by taking 20 point running average.

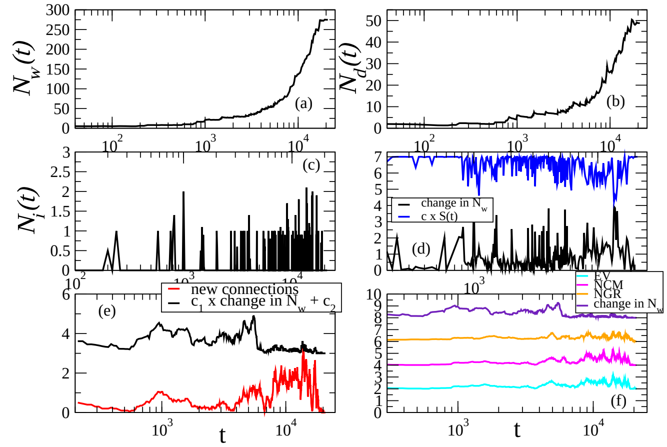

FIG. 8: (Color online) (a) and (b) The temporal evolution of $N_w(t)$ and $N_d(t)$ on time-varying day 22 SGD dataset. The data are averaged over 100 simulation runs. (c) The behavior of the number of inventions of opinions $N_i(t)$ over time. (d) Comparison of $\Delta N_w(t)$ with success rate $S(t)$. (e) Comparison of temporal evolution of $\Delta N_w(t)$ and number of new connections smoothed by taking 20 point running average. (f) Comparison of $\Delta N_w(t)$ with the variance of community sizes (found by NGR, NCM and EV algorithm) evolving over time (the curves are suitably scaled by some constant for the purpose of better visualization). The data are smoothed by taking 20 point running average.

the global quantities as well as different network properties infuencing the game dynamics.

The time evolution of $N_w(t)$ and $N_d(t)$ on the time-varying graph of day 9 (see fig 6(a) and (b)) show a drastically different behavior from the case where these quantities are measured on the static (and composite) counterpart (see fig 5(a) and (b)). The temporal graph shows a slow growth regime followed by a sharp transition, whereas the static counterpart shows steady growth regime followed by a steady fall and finally a long-lasting metastable state (see fig 5(a)). This difference in behavior is due to the fact that in the time-varying case inventions of opinions prevail throughout the dynamics (see fig 6(c)) which prevents the disposal of opinions from the system and hence the memory sizes do not decrease. Further, in fig 6(d) we show how the absolute change in N_w is driven by the success rate; ΔN_w increases with a decrease in $S(t)$ while it decreases with an increase in $S(t)$. Fig 6(e) shows the direct correspondence of the ΔN_w with the new connections. Another interesting property which has an impact on the dynamics is the community size. Indeed the variance of the community sizes relates in a similar way to ΔN_w (see fig 6(f)). Therefore, the continuous inventions, the influx of new connections (causing more failures) and the fact that the opinions get trapped within local neighborhoods together contribute to the steeply rising memory size over the time evolution of the dynamics.

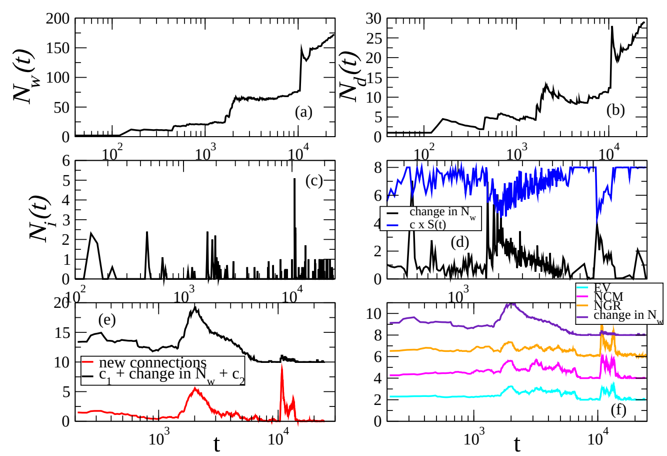

FIG. 9: (Color online) (a) and (b) The temporal evolution of $N_w(t)$ and $N_d(t)$ on time-varying day 26 SGD dataset. The data are averaged over 100 simulation runs. (c) The behavior of the number of inventions of opinions $N_i(t)$ over time. (d) Comparison of $\Delta N_w(t)$ with success rate $S(t)$. (e) Comparison of temporal evolution of $\Delta N_w(t)$ and number of new connections smoothed by taking 20 point running average. (f) Comparison of $\Delta N_w(t)$ with the variance of community sizes (found by NGR, NCM and EV algorithm) evolving over time (the curves are suitably scaled by some constant for the purpose of better visualization). The data are smoothed by taking 20 point running average.

The time-varying networks of 20th, 22nd and 26th day also behave more or less in the same way as for day 9

(see fig 7, fig 8, fig 9). All these time-varying networks typically show a similar slow growth stage followed by a steep rise which is in contrast to their static counterparts (see fig 5) where we found mostly 3 distinct phase of the dynamics - a growth stage, followed by a sharp fall due to series of successful interactions and a meta-stable state due to presence of community structures. This discrepancy in the behavior of $N_w(t)$ and $N_d(t)$ curves is due to the fact that the time-varying networks witness a continuous influx of new agents into the system with inventions happening throughout the evolution and the old agents not playing enough games with the new ones to negotiate and agree upon an opinion.

In fig 10, we show how the frequency of interaction between a pair of individuals predicts the similarity of the opinions among individuals over different instants of time. We measure the similarity of opinions between a pair of individuals by Jaccard Coefficient (JC) of their inventories. It is formally defined as the size of the intersection divided by the size of the union of the inventories i.e., $JC(A_i, A_j) = \frac{|A_i \cap A_j|}{|A_i \cup A_j|}$ where A_i is i^{th} agent's inventory. From all the graphs, it is evident that there is a trend of having higher similarity in opinions with the higher edge-weight where edge-weight reflects the frequency of interactions between a pair till that particular instant of time. Thus, with frequent meetings, individuals tend to share similar opinion. This usually also happens in real-life scenarios where more we meet more similar-opinionated people we become.

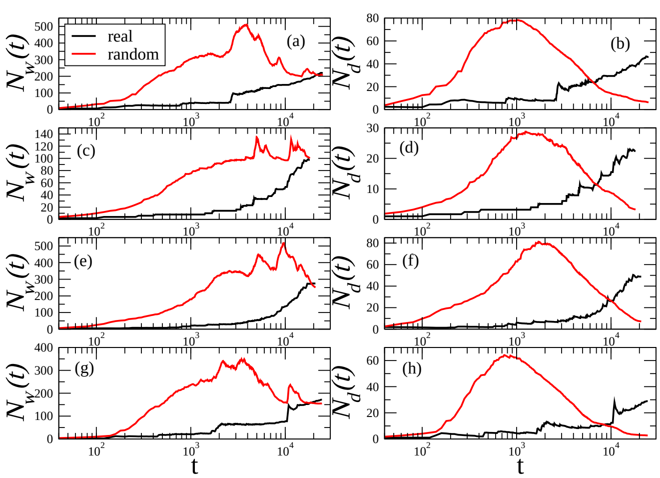

FIG. 11: (Color online) Comparison of the global quantities in the real and the simulated networks. The first row corresponds to the temporal evolution of N_w and N_d for SGD 9th day network. The second, third and fourth row respectively correspond to the temporal evolution of N_w and N_d for SGD 20th, 22nd and 26th day network. The datapoints on the curve are averaged over 100 simulation runs for each of 100 network realizations (in case of the simulated network).

Results from the control experiments: For the purpose of control experiment, we create simulated versions for each of the four time-varying networks by constructing random edges of same number as in the real network in each 20s time interval. We play the naming game on these simulated networks. These types of networks resemble stochastic networks where edges randomly appear or disappear in each epoch. The behavior of the $N_w(t)$ and $N_d(t)$ (see fig 11) are not in the lines of what we observe in the real counterparts. In all the simulated networks, the $N_w(t)$ and $N_d(t)$ behave similarly as in case of static Erdős-Rényi graphs [11].

2. *Results from the HT dataset*

In this section, we consider the second dataset containing dynamic face-to-face interactions among 113 conference attendees. We first study the global behavior of the system through the temporal evolution of three main quantities: the total number $N_w(t)$ of opinions in the system, the number of different opinions $N_d(t)$, and the rate of success $S(t)$.

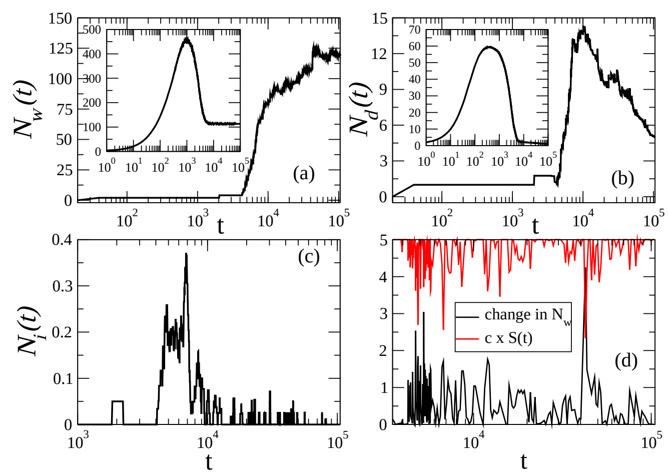

FIG. 12: (Color online) (a) and (b) The temporal evolution of $N_w(t)$ and $N_d(t)$ on time-varying conference network respectively. The insets show the evolution of $N_w(t)$ and $N_d(t)$ on their static counterpart. The data are averaged over 1000 simulation runs. (c) The behavior of the number of inventions of opinions $N_i(t)$ over time. (d) Comparison of $\Delta N_w(t)$ with success rate $S(t)$.

The curve corresponding to $N_w(t)$ shows an initial slow growth followed by a sharp transition and finally reaching a steady growth regime (see fig 12(a)). Note that this result is markedly in contrast to what would have been observed if the games were played on the composite network constructed at the end of the conference (see fig 12(a) inset). In fact, this result is in contrast to most of the other results that have been reported in the literature so far indicating that the time-varying nature of the underlying societal structure with new connections being formed, old connections being dropped and agents

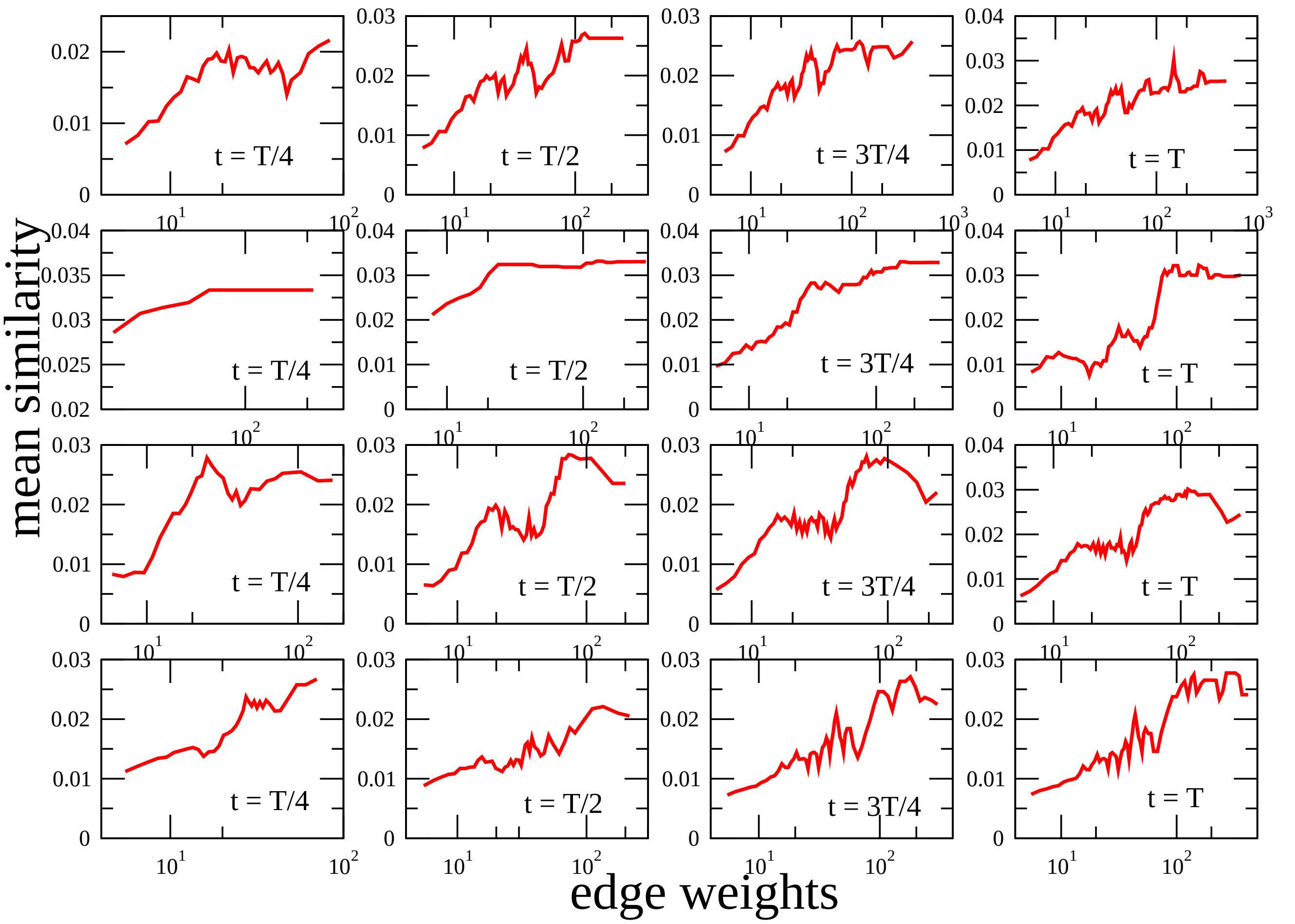

FIG. 10: Comparison of mean similarity with edge weights. The graphs on the first row show the similarity results on SGD 9th day for four different instances of time at $t = T/4$, $t = T/2$, $t = 3T/4$ and $t = T$ where T is the total time. Similarly, rows 2, 3 and 4 show similarity results for SGD 20th, 22nd and 26th day respectively at four different time instances - $t = T/4$, $t = T/2$, $t = 3T/4$ and $t = T$. The curves are smoothed by taking 10 point running average.

entering, leaving and re-entering the system has a strong impact on the emergent pattern of opinion formation. Similar trends are also observed for $N_d(t)$ - initially a slow growth followed by a sharp transition reaching a peak and finally a drop, however, no way close to 1 (see fig 12(b)). The inset in fig 12(b) shows the evolution of N_d if the games were played on the composite network finally obtained.

Initially, as time proceeds, new individuals join the network that increases the number of inventions of new opinions (see fig 12(c)) thus causing a rise in both $N_w(t)$ and $N_d(t)$. However, later on new inventions stop (fig 12(c)) as the players joining late are less compared to the number that have already joined and are therefore rarely chosen as speakers thus inhibiting new inventions. Hence, $N_d(t)$ is found to drop in the later stage of the dynamics although pointing to a clear existence of multiple opinions. In contrast, $N_w(t)$ doesn't drop because although new opinions are not formed, old opinions trapped in different groups do not get disposed off the system.

Further, in fig 12(d) we show how the absolute change in N_w is driven by the rate of success of agents; ΔN_w increases with a decrease in $S(t)$ while it decreases with an increase in $S(t)$. Finally, an important analysis that is required to complete the picture centers around the precise reason for the steady growth in N_w in the final regime of the dynamics. We attempt to provide a plausible explanation for this through a series of results reported in fig 13.

In fig 13(a), we present the fraction of agents having 0, 1, 2 and more opinions in their inventories. Clearly, with the evolution of system, the fraction of agents with inventory size 0 diminishes; fraction of agents with size 1 increase steadily while that with size 2 is roughly stable; even larger size inventories appear only rarely in the course of the evolution. In addition, we observe that ΔN_w has a direct correspondence with the number of new connections acquired by the network at each timestep (see fig 13(b)). These new connections trigger an increase in failure events, thereby increasing N_w; at the

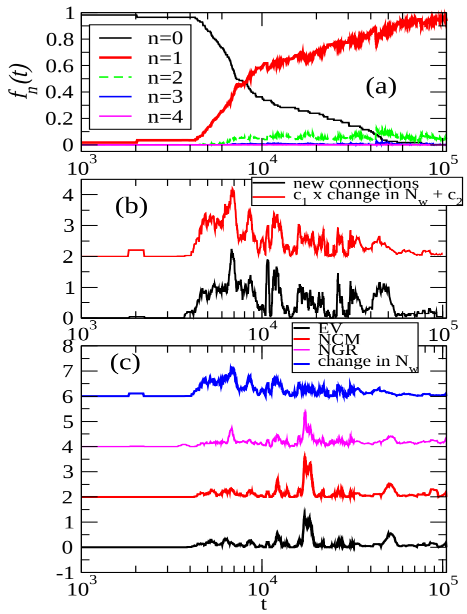

FIG. 13: (Color online) (a) Evolution of inventory sizes n ($n = 0, 1...$). $f_n(t)$ is the fraction of agents whose inventory size is n at time t. (b) Comparison of temporal evolution of $\Delta N_w(t)$ and number of new connections smoothed by 20 point running average. (c) Comparison of $\Delta N_w(t)$ with the variance of community sizes (found by NGR, NCM and EV algorithm) evolving over time (the curves are suitably scaled by some constant for the purpose of better visualization). The data are smoothed by taking 20-point running average.

same time success events cannot reduce N_w since in most cases the inventory sizes of the agents are already very low (~ 1) and most of these success events are actually again "success with no outcome". This last observation indicates that there should be an inherent community structure in this time-varying network and this is made apparent through fig 13(c) where we report the variance of the the size of the communities (using three different algorithms as in the previous cases) and show that this is highly correlated to ΔN_w. In summary, the presence of community structure coupled with a continuous influx of new connections (leading to late-stage failures in the system) together lead to the steady growth of N_w in its final regime of evolution.

In fig 14, we present mean similarity of opinions among individuals with edge weights in different time instances. In all the instances, there is a positive correlation of having similar opinions with frequency of interactions i.e., higher the frequency of interactions (edge-weight), higher is the similarity in opinion.

Results from control experiments : For this HT dataset also, we create simulated networks where at each 20s time interval, we construct m number of random edges with m being the count of edges that appeared on that time interval in the real network. We observe the two most important observables $N_w(t)$ and $N_d(t)$ by playing naming

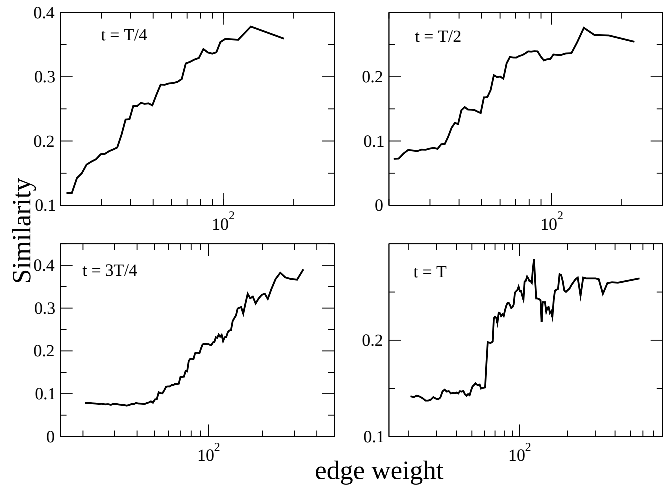

FIG. 14: Comparison of mean similarity with edge weights on HT dataset. The mean similarity of opinions with edge weights are shown at different instances of time (a) at $t = T/4$, (b) $t = T/2$, (c) $t = 3T/4$ and (d) $t = T$ where T is the total time. The curves are smoothed by taking 40 point running average.

game on these simulated networks. Both these quantities show a different behavior from its real counterpart. The $N_w(t)$ and $N_d(t)$ in the simulated networks show distinct two regions - a steady growth and then a fall whereas the $N_w(t)$ curve in the real network show a slow growth zone followed by a sharp transition and finally a zone of steady growth. The simulated networks tend to behave as standard Erdős-Rényi graphs.

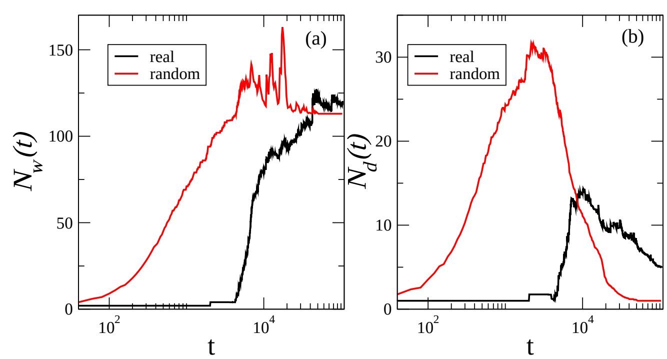

FIG. 15: (Color online) Comparison of the global quantities in the real and the simulated networks. Temporal evolution of (a) N_w, (b) N_d for HT real and simulated dataset. The datapoints on the curve are averaged over 100 simulation runs for each of 100 network realizations (in case of simulated network).

VI. CONCLUSIONS AND FUTURE WORK

In this paper, we studied the Naming Game as a model of opinion formation on the time-varying social networks. Some of our key observations are:

(a) While considering composite snapshots accumulated over a certain period (e.g., 69 instances of SG dataset with each instance being an accumulation of snapshots) both the maximum memory and the time to reach this memory peak scale as population size (N); however, the time to reach the consensus strongly depends on the presence of community structure (rather than a straight-forward $N^{1.4}$ scaling);

(b) While considering the time-evolution of the network in perfect synchronization with the steps of the game (e.g., SGD and HT dataset) we observe that the emergent behavior of the most important observables (i.e., $N_w(t)$ and $N_d(t)$) have a nature that is markedly in contrast to what has been reported so far in the literature thus indicating the strong influence of the underlying societal structure on the dynamics of opinion formation. While in case of SGD, we observe that new inventions along with a continuous influx of new agents keeps both $N_w(t)$ and $N_d(t)$ sharply growing, in case of HT, successful interactions among older agents cause inventions to stop (hence a fall in $N_d(t)$) although late-stage failures continue to exist due to influx of new agents thus contributing to a steady growth of $N_w(t)$ in the final phase of the dynamics. The fall of $N_d(t)$ curve is not observed in case of SGD possibly because in this case the games are played over a shorter span of time (1 day) in comparison to HT where the games are played over 2.5 days so that enough successful interactions could be realized.

There are quite a few interesting directions that can be explored in the future. One such direction could be to incorporate the dominance index of the agents into the model. Not all actors in a society are equally dominant; while some of the actors are more opinionated and dominant the others might be more passive. This characteristic property can be incorporated into the model by ranking those agents that are more successful in their past interactions as more dominant. In this setting, it would be interesting to investigate the scaling relations most naturally under the constraints that the dominant agents are allowed to speak more. Another direction could be to investigate the effect of the flexibility of the agents in adapting to new opinions (traditionally modeled by a system parameter β that encodes the probability with which the agents update their inventories in case of successful interactions [4]) when they are embedded on time-varying networks. Finally, a thorough analytical estimate of the important dynamical quantities reported only through empirical evidence here is needed to have a "clear-cut" understanding of the emergent behavior of the system.

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